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## IMPACT OF SPHERICAL SHELLS AGAINST THE SURFACE OF A LIQUID

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ABSTRACT. Problems involving the impact of flat and elastic bodies against a liquid began to be intensely investigated as far back as the 1930's in connection with designs of hydroplanes during landing and ship impact against waves [1-3]. During an impact of a body against a liquid, the values and the character of the distribution of the hydrodynamic load on the surface of the body are influenced by many factors. It is difficult to take all of these factors into consideration in view of the nonlinearity of the boundary conditions on the free surface, the presence of current filaments, and splashes leading to discontinuous solutions.

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At the present time, the water impact of rigid bodies has been investigated in greatest detail. Problems in which the hydroelastic interaction between a body and a liquid was taken into account have been investigated drawing upon the concepts introduced in the 1930's by Wagner essentially for flat-keeled bodies [3-5]. A detailed bibliography of the problem of impact of bodies against a liquid is given in the survey [6].

An analysis is made here of the vertical impact of a thin, mildly sloping spherical shell against the surface of an ideally incompressible liquid. The contour of the shell leans against an elastic rib which is attached to a rigid body of mass  $M_0$ . The mass of the rigid body,  $M_0$ , exceeds many times the mass of the shell,  $m_0$ . It is assumed that the initial impact velocity  $v_0$  is small compared to the velocity of sound  $c$  in the liquid.

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\*Numbers in the margin indicate the pagination in the original foreign text.

1. Below we shall limit ourselves to a discussion of only axially symmetric deformations of the shell. Then, for certain ratios of the parameters of the shell and the support rib [7], the equation of motion of the shell in terms of the normal bending  $w_1$  and the boundary conditions can be written in a particularly simple form

$$\frac{\partial^2 w}{\partial \tau^2} = -A \nabla^2 \nabla^2 w + B \nabla^2 w - \gamma w + \frac{1}{k} \frac{dV}{d\tau} + D p^* \quad (1)$$

$$w = \nabla^2 w = 0 \quad \text{for } \alpha = 1$$

Here

$$w = w_1 / h, \quad k = h / R, \quad \tau = ct / R, \quad \gamma = E / \rho_0 c^2, \quad p^* = p / \rho c^2$$

$$A = \gamma k^2 / 12 \varphi_0^4 (1 - \nu^2), \quad B = N_0 \gamma h / E R_0^2 k^2, \quad D = 1 / \eta k^2, \quad \eta = \rho_0 / \rho$$

$$V = v / c, \quad \varphi_0 = R_0 / R, \quad \alpha = r / R_0, \quad \nabla^2 = \partial^2 / \partial \alpha^2 + 1 / \alpha \partial / \partial \alpha$$

where  $h, R$  are the thickness and the radius of curvature of the shell;  $\nu, E, \rho_0$  are the Poisson ratio, elasticity modulus, and density of shell material, respectively;  $\rho, c$  are the density of the liquid and the velocity of sound in it;  $R_0$  is the radius of the support rib;  $t$  is the time;  $r$  is the radius vector of the cylindrical coordinate system;  $p$  is the hydrodynamic pressure;  $v$  is the shell motion velocity as a rigid body;  $N_0$  is the constant initial stress in the shell.

In the derivation of Equation (1), we have not taken into account the forces of inertia in the middle surface of the shell and the projection of the inertial force due to the motion of the shell as a rigid body on the tangent to the shell contour. A positive bending value is in the direction of the inner normal. Initially  $w = \partial w / \partial \tau = 0$  for  $\tau = 0$ .

2. To determine the hydrodynamic loads in the initial stage of the interaction of the shell with the liquid, we shall use the same concepts that Wagner used for the wedge in the two-dimensional case [1]. For small

depths of immersion, the flow around the body of revolution will be analogous to the flow in front of an equivalent disk of radius  $b(t)$  which moves forward in translational motion.

Then the velocity potential for a perturbed motion of the liquid,  $\phi = \phi_1 + \phi_2$ , (assuming it exists) may be represented in the form ( $\mu, \xi$  are the elliptical coordinates)

$$\phi_1 = \frac{2v}{\pi} b\mu(1 - \xi \operatorname{arctg} \xi), \quad \phi_2 = \sum_{n=0}^{\infty} B_n P_{2n+1}(\mu) Q_{2n+1}(i\xi)$$

$$B_n = -b \left[ \frac{\partial}{\partial \xi} Q_{2n+1}(i\xi) \right]_{\xi=0}^{-1} \int_0^1 \mu \frac{\partial w_1}{\partial t} P_{2n+1}(\mu) d\mu \quad (2)$$

Here  $P_{2n+1}(\mu)$ ,  $Q_{2n+1}(i\xi)$  are Legendre polynomials of first and second kind;  $\mu=0$  corresponds to a free surface of liquid;  $\xi=0$  corresponds to the surface of the disk.

The potential  $\phi_1$  corresponds to a translational motion of the disk with velocity  $v$ , and the potential  $\phi_2$  is due to the presence of an additional velocity field  $\partial w / \partial t$ , which is caused by a deformation of the shell [8,9].

After conversion to dimensionless coordinates, we obtain the following on the basis of the Cauchy-Lagrange relation

$$p^* = p_1^* + p_2^*, \quad p_1^* = \frac{2}{\pi} \left[ \frac{V^2 a}{\varphi_0 u \sqrt{\beta^2 - \alpha^2}} + \varphi_0 \sqrt{\beta^2 - \alpha^2} \frac{dV}{d\tau} \right]$$

$$p_2^* = -\frac{k}{2\beta^2} \sum \gamma_n \left\{ \frac{V}{u} \left[ P_{2n+1}(\mu) + \frac{1-\mu^2}{\mu} \frac{\partial}{\partial \mu} P_{2n+1}(\mu) \right] \times \right.$$

$$\left. \times \int_0^1 \alpha \frac{\partial w}{\partial \tau} P_{2n+1}(\mu) d\alpha + a P_{2n+1}(\mu) \int_0^1 \alpha \frac{\partial^2 w}{\partial \tau^2} P_{2n+1}(\mu) d\alpha \right\} \quad (3)$$

where

$$f_n = \frac{4n+3}{\pi} \left[ \frac{n! 2^{2n+1}}{(2n+1)!} \right]^2, \quad \mu = \frac{\sqrt{\beta^2 - \alpha^2}}{\beta}$$

$$a = \frac{b}{R}, \quad u = \frac{V}{da/d\tau}, \quad \beta = \frac{b}{R_0}$$

The function  $u(\tau)$  has in this case been introduced formally by analogy with the impact of rigid bodies, and no longer has universal meaning as it did in the case of impact of rigid bodies ( $u(\tau)$  must be found during the solution).

In deriving Equations (3), we have neglected certain terms in the Cauchy-Lagrange integral, the terms being proportional to the square of the total velocity of motion of the liquid. The role of these terms increases with an increase in the depth of immersion.

The greatest difficulty in this approach consists in determining the radius of the wetted surface,  $b(t)$ . It can be found from the corresponding integral equation as in the case of a vertical impact of a flat-keeled elastic body [5].

Here, to simplify the problem, to determine  $b(t)$  we shall consider the vertical impact of an equivalent mechanical system consisting of two rigid bodies of masses  $M_0$  and  $m_0$  connected by an elastic spring. Contact with the liquid is made by the body  $m_0$  whose mass and the form of the impact surface correspond to the mass and form of the rigid shell. The motion of the system will be described by the following equations [3]:

$$x_1'' + \omega x(x_1 - x_2) - \vartheta = 0, \quad (1+m)x_2'' + m x_2'' - \omega(x_1 - x_2) - \vartheta = 0 \quad (4)$$

and

$$x_1 = y_1 / R, \quad x_2 = y_2 / R, \quad \omega = \omega_0 R^2 / c^2 m_0$$

$$\vartheta = gR / c^2, \quad m = 4\rho b^3 / 3m_0, \quad \kappa = m_0 / M_0$$

Here  $y_1$  is the displacement of the body  $M_0$ ;  $y_2$  is the displacement of the body  $m_0$ ;  $\omega_0$  is the stiffness of the spring;  $g$  is the acceleration of gravity.

The stiffness of the spring,  $\omega_0$ , may be determined experimentally or theoretically.

In this case, the function  $u = x_2^*/a^*$  is defined by the expression [10]

$$u = \frac{1+a^2}{4a^2} \ln \frac{1+a}{1-a} - \frac{1}{2a} \quad (5)$$

To system (4) we must add the initial conditions

$$x_1 = x_2 = 0, \quad \dot{x}_1 = \dot{x}_2 = V_0 \quad \text{for } \tau = 0 \quad (6)$$

System (4) with (5) and (6) was integrated numerically using the Runge-Kutta method (the stiffness  $\omega_0$  was determined in terms of the frequency of the first oscillatory mode of the shell). By solving this system, we determined the functions  $a(\tau)$ ,  $u(\tau)$ ,  $V(\tau)$  and  $V'(\tau)$ . The data thus obtained were used subsequently to solve Equation (1).

3. For the particular case discussed here, it is convenient to represent the displacement of the shell,  $w(a, \tau)$ , in the form

$$w = \sum_i \Theta_i J_0(\xi_i a) \quad (7)$$

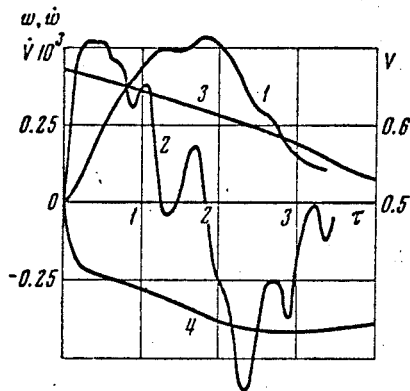
where  $J_0(\xi_i a)$  is the zero-order Bessel function, and  $\xi_i$  are the roots of the equation  $J_0(\xi_i) = 0$ .

Expanding the external load into a series in the corresponding Bessel functions, after using the I. G. Bubnov procedure [11] we shall obtain a system of ordinary differential equations in the generalized coordinates,

$\Theta_i(\tau)$ , which was integrated numerically by the Runge-Kutta method with the following values of the characteristic parameters of the shell and the medium:

$$k = 0.0255, \quad \varphi_0 = 0.42, \quad \gamma = 3.91, \quad \eta = 0.428, \quad V_0 = 6.66 \cdot 10^{-3}, \quad \nu = 0.3, \quad \kappa = 0.07.$$

Calculations were done for  $i = 1 - 7$ ,  $n = 0 - 4$ .



The figure shows the variation of the total bending  $w$  (curve 1) and the velocity (curve 2) at the center of the panel. In the same figure, curve 3 characterizes the variation with time of the velocity of the entire system as a rigid body,  $V$ , and curve 4 corresponds to  $V'$ .

The greatest contribution to the total bending is made by the first four terms of the series.

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